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Weak Matrix Elements in QCD: Large N_c Expansions

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Abstract

A consistent formulation of the $K \to \pi\pi$ weak decay amplitudes is presented within the context of quantum chromodynamics. Using the large N_c limit, the perturbative short distance treatment is combined with a bosonized version of QCD for the large distance contributions to the weak matrix elements. The calculated large distance contributions provide a further enhancement of the $\Delta I=1/2$ amplitudes and suppression of the $\Delta I=3/2$ amplitudes as indicated by the data. The large distance contributions to the $\Delta S=2$ amplitudes for K^0-$\overline{K^0}$ mixing are estimated and a consistent value of the B parameter is obtained.

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The large enhancements observed for the $\Delta I = 1/2$ weak decay amplitudes have long been a puzzle for the standard model. Partial explanations of the observed effects have involved short distance QCD evolution of the relevant weak operators [1] and the Penguin diagram contributions [2]. These short distance effects are not sufficient to explain the full enhancements seen in the data. The recent efforts have therefore focused on the long distance contributions to the weak matrix elements. Preliminary results of detailed lattice computations of these matrix elements are reported in contributions to this meeting [3]. I will present an alternative method of calculation which makes use of a bosonization of QCD which should be valid in the large N_c limit [4].

The conventional treatment of the weak decay processes uses an effective weak Hamiltonian where the short distance effects of the W-boson propagator, heavy quarks, and perturbative QCD have been absorbed in calculable coefficient functions. The weak Hamiltonian can be represented as

$$H^{\Delta S=1} = -(G_F/\sqrt{2})\cdot s_1 \cdot c_1 \cdot c_3 \cdot \Sigma \{z_i(\mu) + \tau \cdot y_i(\mu)\} \cdot \mathbf{Q}_i(\mu) \tag{1}$$

where τ is given by the KM angles, $\tau = (s_2)^2 + (s_2 \cdot s_3 \cdot c_2 / c_1 \cdot c_3) \cdot e^{-i\cdot\delta}$ and the dominant operators for low energy matrix elements are given by

$$\mathbf{Q}_1 = (\bar{s}\, d)_{V-A} \cdot (\bar{u}\, u)_{V-A}$$

$$\mathbf{Q}_2 = (\bar{s}\, u)_{V-A} \cdot (\bar{u}\, d)_{V-A} \tag{2}$$

$$\mathbf{Q}_6 = -8 \cdot \mathbf{\Sigma}_q\, (\bar{s}_L\, q_R) \cdot (\bar{q}_R\, d_L).$$

The coefficient functions $z_i(\mu)$ and $y_i(\mu)$ have a strong dependence on the normalization scale, μ, at low energy due to the QCD corrections.

The amplitudes for $K \to \pi\pi$ decay are given by the matrix elements of the weak Hamiltonian given in Eq(1),

$$\langle \pi\pi | H^{\Delta S=1} | K \rangle = -(G_F/\sqrt{2})\cdot s_1 \cdot c_1 \cdot c_3 \cdot \mathbf{\Sigma}\{z_i(\mu) + \tau \cdot y_i(\mu)\} \cdot \langle \pi\pi | \mathbf{Q}_i(\mu) | K \rangle \tag{3}$$

It is clear that the μ dependence of the coefficient functions must be cancelled by the μ dependence of the operator matrix elements [5]. Many different methods have been used to estimate these matrix elements including factorization (or vacuum insertion), bag models, sum rules, chiral quark models ($\Lambda_{chiral} >> \Lambda_{QCD}$), and the lattice methods mentioned above [3]. None of these methods, except perhaps the lattice, have demonstrated the explicit cancellation of the μ dependence and hence a consistent matching of the short and long distance effects. I will describe an alternate method which makes use of the large N_C limit of QCD where N_C is the number of colors.

The large N_C expansion of QCD is defined by the limit, $N_C \to \infty$ with $\alpha_S \cdot N_C$ held fixed. In this limit the leading contributions come from planar diagrams with the least number of quark loops. Nonplanar diagrams and the insertion of additional quark loops are suppressed by powers of $1/N_C$. This planar limit of QCD produces a kind of hadronic string theory which has yet to be solved exactly. The order of perturbative QCD corrections is easily computed with the current vertex corrections being $O(1)$ and the usual weak evolution and penguin contributions being $O(1/N_C)$. Hence, only the factorized matrix elements contribute in the leading N_C limit, and these are simply evaluated as the matrix elements of the weak currents are known. All nonfactorizing contributions are formally of order $1/N_C$. At short distance, this expansion is quite accurate even for $N_C=3$ [4]. However, there can be effects which compensate the $1/N_C$ suppression. The evolution of the weak operators is not suppressed if the ratio M_W/μ is sufficiently large (the long evolution compensates for a small velocity), and the penguin suppression can be compensated by large matrix elements. In leading order of perturbative QCD, the μ dependence of the weak matrix elements is given by

$$\mu\partial_\mu\langle\pi\pi|\mathbf{Q}_1(\mu)|K\rangle = -(3/2\pi)\cdot\alpha_S(\mu)\cdot\langle\pi\pi|\mathbf{Q}_2(\mu)|K\rangle$$

$$\mu\partial_\mu\langle\pi\pi|\mathbf{Q}_2(\mu)|K\rangle = -(3/2\pi)\cdot\alpha_S(\mu)\cdot\langle\pi\pi|\mathbf{Q}_1(\mu)|K\rangle - \overline{\gamma}_{26}(\mu)\cdot\alpha_S(\mu)\cdot\langle\pi\pi|\mathbf{Q}_6(\mu)|K\rangle \qquad (4)$$

where all contributions are formally $O(1/N_C)$ since α_S is $O(1/N_C)$.

The general factorized planar diagrams as shown in Fig.(1a) are leading

N_c while the leading $1/N_c$ corrections are given by diagrams as shown in Fig.(1b). Any number of planar gluons may be added to the diagrams without changing the order of the large N_c expansion. In this large N_c or string limit, QCD reduces to a theory of weakly interacting meson resonances. Therefore

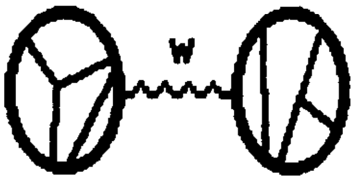

Figure 1a: Factorized, leading N_c amplitude.

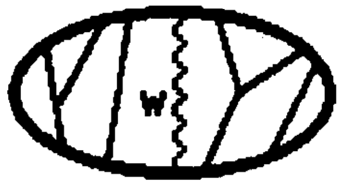

Figure 1b: Nonfactorized, planar amplitude, $O(1/N_c)$.

we should be able to obtain a dual meson representation of QCD with the dynamics provided by the interactions of a complete set of meson resonances, π, ρ, f, ... All local operators ($\overline{\Psi}\gamma_\mu\Psi$, $\overline{\Psi}\Psi$, etc) should also have a meson representation. The meson theory is fully equivalent to the quark theory in the large N_c limit and can be considered as a complete bosonization of the quark theory such as is commonly done in two dimensional field theories. However, the large N_c limit provides an essential simplification as the leading quark amplitudes are completely reproduced by the tree amplitudes for the meson theory. The $1/N_c$ expansion corresponds to the meson loop expansion. The meson theory including all meson resonances and loops is therefore equivalent to complete quark theory which includes the sum of both perturbative and nonperturbative QCD.

Of course, in the meson representation short distance physics is exceedingly complex requiring many higher mass resonance states and knowledge of their complex interactions. On the other hand, the quark theory is quite simple and perturbative at short distance while the long distance physics is dominated by the nonperturbative aspects of QCD. We will try to take advantage of the simple aspects of both representations of QCD by introducing a scale μ to separate the short and long distance physics. For short distances, $Q^2>\mu^2$, we will use the quark theory and for long distances, $Q^2<\mu^2$, we will use the meson theory. Since both theories provide a complete description of the physics, nothing should depend on the scale μ. However, if we consider extreme limits for the scale μ, there can be important simplifications. At short distance, the quarks are accurately described by perturbative QCD, and the nonperturbative aspects are small. At large distance, the meson theory should be truncated on the lightest meson states, and in the extreme limit it should be sufficient to keep only the pseudoscalar mesons states with their dynamics specified by unique chiral Lagrangians. A matching between these two extreme pictures may be possible if a sharp transition exists between perturbative and nonperturbative QCD at the common scale, $\Lambda_{QCD} \approx \Lambda_{chiral} \approx m_{constituent\ quark}$. Hence, we will use the truncated meson theory to compute the meson matrix elements which must be matched to the quark theory at a reasonable scale for both theories. We find this scale to be about 0.6 - 0.8 GeV in our calculations.

The truncated meson theory is described by the following chiral Lagrangian,

$$L_{tr} = (f^2_\pi/4)\cdot[\ tr\{D_\mu U\cdot D^\mu U^+\} + r\cdot tr\{mU+U^+m\} - (r/\Lambda^2_\chi)\cdot tr\{mD^2U+D^2U^+m\}\ ] \quad (5)$$

where $U = U(\pi)$ is the unitary chiral matrix, $D_\mu U = \partial_\mu U + i\cdot W_\mu U$ is the gauge covariant derivative, and m is the quark mass matrix. Using the action of Eq.(5), we can obtain a consistent formulation of the currents,

$$(\bar{q}^j\gamma_\mu q_i)_{V-A} = i\cdot(f^2_\pi/2)\cdot\{(\partial_\mu U)U^+ - U(\partial_\mu U^+) - (r/\Lambda^2_\chi)\cdot[m(\partial_\mu U^+) - (\partial_\mu U)m]\}_{ij} \quad (6)$$

and densities,

$$(\bar{q}_R{}^j q_{L\,i}) = -\,(f^2_\pi/4)\cdot r\cdot[\ U - (1/\Lambda^2_\chi)\cdot\partial^2 U\ ]_{ij} \quad (7)$$

$\Lambda_\chi \approx 1$ GeV is the scale of the higher derivative terms in the action of Eq.(5) which are expected to exist in any truncated theory and are needed for the a consistent evaluation of the penguin matrix elements [6]. We emphasize that L_{tr} is not the usual effective tree Lagrangian, but loops must be included to a consistent order in the $1/N_c$ expansion.

There have been similar approaches using chiral Lagrangians which differ from the present approach [4] in essential ways The usual factorized procedure [6] is equivalent to the use of chiral Lagrangians in tree approximation which neglects calculable $1/N_c$ corrections and ignores all nonfactorizing contributions. Chiral perturbation theory [7] includes calculable loop effects but only those which are higher order in the meson masses or momenta. However, these two approaches are not consistent with the weak operators defined in the quark picture and cannot be used with the coefficient functions given in Eq.(3) to obtain the complete weak decay amplitudes. Another approach [8] uses the spontaneous chiral symmetry breaking in the quark picture to introduce the meson interactions and incorporate more of the long distance physics in weak matrix elements.

The essential difference of our method [4] concerns the use of a physical cutoff scale, $M \approx \mu$, to separate the short and long distance physics contributions. The meson theory must be computed with this cutoff, M. This cutoff is essential in the truncated meson theory because of the quadratic divergences. It is this dependence on the cutoff, particularly the quadratic divergences which allows us to match the short distance quark picture. A more complete meson theory including higher meson resonances might not have the quadratic divergences of the truncated theory but would still require the systematic introduction of the scale M.

Our truncated chiral Lagrangian depends on parameters, $f_\pi(M)$ and Λ_χ which can be determined in terms of the physical meson decay consants, F_π and F_K, through $O(1/N_c)$. We obtain

$$f^2_\pi(M) = F^2_\pi - 2\cdot(f_\pi \cdot m_\pi/\Lambda_\chi)^2 + 2\cdot I_2(m_\pi) + 2\cdot I_2(m_K) \qquad (8)$$

with $I_2(m) = (4\pi)^{-2} \cdot [M^2 - m^2 \cdot \log(1+M^2/m^2)]$

$$F_K/F_\pi = 1 + (m^2_K - m^2_\pi)/\Lambda^2_\chi - (3/8\cdot f^2_\pi)\cdot[2\cdot I_2(m_K) + 3\cdot I_2(m_8) - 5\cdot I_2(m_\pi)] \qquad (9)$$

From Eq.(8) we see that f^2_π has a quadratically divergent renormalization where $1/f^2_\pi$ is the effective meson coupling constant at scale M. The result for F_K/F_π agrees with the usual chiral perturbation theory results [6,7]. The parameter $\Lambda_\chi \approx 1$ GeV using Eq.(9) and ≈ 0.9 GeV if only the leading N_c terms are kept.

To compute the evolution of the $K \to \pi\pi$ weak matrix elements, we must include the full one loop contributions which contribute nonfactorizing matrix elements of the weak operators, $\mathbf{Q}_1$ and $\mathbf{Q}_2$. We have computed these matrix elements in Ref.[4]. From the structure of these matrix elements, we can abstract the effective evolution properties of the weak operators. We relate the operators defined at scale M to the operators defined at zero (or trees),

$$\begin{aligned}\delta \mathbf{Q}_1 &= (1/f^2_\pi)\cdot F_1(M)\cdot \mathbf{Q}_2(0)\\ \delta \mathbf{Q}_2 &= (1/f^2_\pi)\cdot F_1(M)\cdot \mathbf{Q}_2(0) + (1/f^2_\pi)\cdot F_2(M)\cdot (\mathbf{Q}_2(0) - \mathbf{Q}_1(0))\end{aligned} \tag{10}$$

with

$$\begin{aligned}F_1(M) &= -2\cdot M^2/(4\pi)^2 + \cdots\\ F_2(M) &= M^2/(4\pi)^2 + \cdots\end{aligned} \tag{11}$$

The running of these matrix elements as seen through the M^2 dependence of Eq.(11) has the correct structure to match the short distance quark evolution. Not only is the $\mathbf{Q}_1$-$\mathbf{Q}_2$ mixing properly represented but there is also a term in $\delta \mathbf{Q}_2$ which represents the continued evolution of the penguin diagrams. To evaluate fully the penguin contributions to this order in the $1/N_c$ expansion, we can use the leading N_c expressions for the quark densities given in Eq.(7) and the $\mathbf{Q}_6$ operator given in Eq.(2),

$$\begin{aligned}\mathbf{Q}_6 &= -8\cdot \Sigma_q\,(\overline{s}_L\, q_R)\cdot(\overline{q}_R\, d_L) = 4\cdot f^4_\pi\cdot (r/\Lambda_\chi)^2\cdot\{\partial U\cdot \partial U^+\}_{ds}\\ &= 4\cdot (r/\Lambda_\chi)^2\cdot[\mathbf{Q}_2(0) - \mathbf{Q}_1(0)]\end{aligned} \tag{12}$$

From this expression for the matrix elements of $\mathbf{Q}_6$ and the expressions for $\mathbf{Q}_1$ and $\mathbf{Q}_2$ given in Eq.(11) we can evaluate the full $K\to\pi\pi$ decay amplitudes. The details are given in Ref.[4]. The summary of the quark and meson evolution is given in Table I. From the results given in Table I it is clear that the full evolution of the weak matrix elements provides a significant enhancement of the $\Delta I=1/2$ amplitude and sufficient suppression of the $\Delta I=3/2$ amplitudes. It is also clear that the penguin contribution does not dominate our results which is consistent with the lattice calculations [3].

The use of the large N_c limit to obtain a boson representation of QCD gives a consistent method for evaluating weak decay amplitudes. Despite the severe truncation of the meson theory, we find proper the scale dependence of the meson matrix elements achieved when the quadratic divergent terms are properly included. Despite the crude approximations used, the combined quark and meson calculations give a large enhancement of the of the $\Delta I=1/2$ and a similar suppression of the $\Delta I=3/2$ which seems consistent with the data. The procedure can certainly be improved through a less severe truncation of the meson theory by including vector mesons, etc which could allow extrapolation of the meson theory to higher scales. The leading log quark calculation could also be improved by computing the higher orders in perturbative QCD. The matching conditions between the quark and meson contributions could be done in a more sophisticated manner than by simply equating the normalization scale of the quark operators with the cutoff of the meson theory as we have done above.

Finally we have also applied these methods to the CP violating amplitudes in Ref.[9]. $\epsilon' \propto \mathrm{Im}\, A_0$ comes mostly from the penguin contribution associated with the top quark which is dominated by short distance effects. $\epsilon \propto \mathrm{Im}\, M_{12}$ comes from the box diagram which is sensitive to the long distance effects. Using the meson theory described above, the B parameter can be computed with the result, $B \approx 0.66 \pm 0.1$ which can be compared to 0.75 for the leading N_c calculation. It is again the inclusion of the quadratic divergent terms in the meson theory which are responsible for the stable behavior of the large N_c expansion in contrast to the results obtained from chiral perturbation theory.

Table I: The evolution of the weak decay amplitudes for $K \to \pi\pi$.

$A(K \to \pi\pi)$ 10^{-8} GeV	μ GeV	m_s GeV	$K^0(\Delta I=1/2)$	$K^0(\Delta I=3/2)$	$K^+(\Delta I=3/2)$
DATA			38.2	1.0	1.8
Initial	m_W	-	3.9	3.9	3.9
w/ quark evlolution only	m_c	-	6.8	2.9	2.9
	0.8	-	8.8	2.6	2.6
w/ quark evolution and penguin	0.8	0.125	19.0	2.6	2.6
	0.8	0.150	16.0	2.6	2.6
w/quark, penguin, and meson evolution	.6-.8	0.125	30.0	1.6	1.6
	.6-.8	0.150	27.0	1.6	1.6
	.6-.8	0.175	25.0	1.6	1.6

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